

Name: \_\_\_\_\_ Instructor: \_\_\_\_\_

---

**This exam has three parts. Please read carefully the directions for each part. All problems are of equal point value. No notes, books, cell phones, or any device that can connect to the internet is allowed.**

---

## **PART ONE**

You must complete this portion of the test **without** using a calculator. For full credit you must show all appropriate work and clearly indicate your answers. After you have finished part one, your instructor will give you the remaining parts of the exam.

When simplifying answers, it is **not** necessary to rationalize the denominator.

---

- 1) **Given that  $\alpha$  is an angle in standard position whose terminal side contains the point  $(3, -2)$ , provide the exact value of each trigonometric function. It is not necessary to rationalize denominators.**

$$\sin \alpha =$$

$$\cos \alpha =$$

$$\tan \alpha =$$

$$\csc \alpha =$$

$$\sec \alpha =$$

$$\cot \alpha =$$

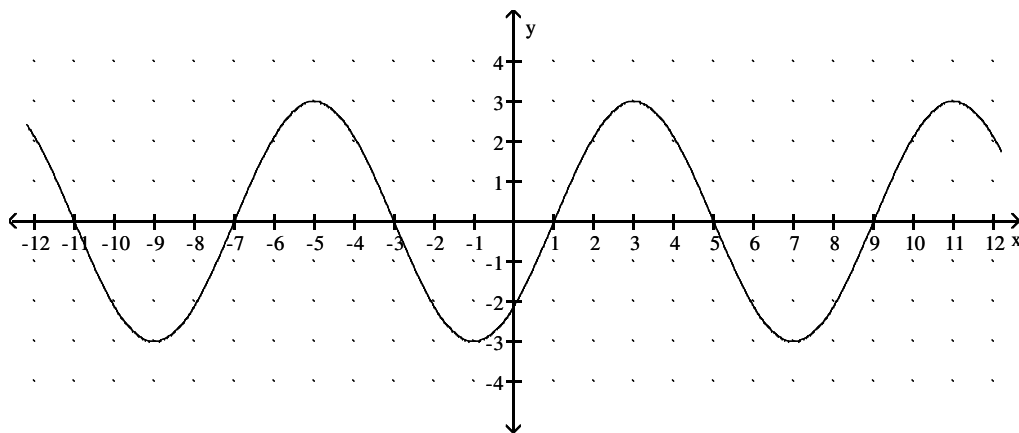
**Find all real numbers in the interval  $[0, 2\pi)$  that satisfy the equation.**

2)  $5 \cos x = 2 \cos^2 x + 2$

**Solve the problem.**

- 3) If  $\cos \alpha = \frac{2}{3}$  and  $\sin \beta = \frac{1}{5}$ , where  $0 \leq \alpha \leq \frac{\pi}{2}$  and  $\frac{\pi}{2} \leq \beta \leq \pi$ , then find  $\cos(\alpha - \beta)$ .

- 4) Write an equation of the function in the graph in the form  $y = A \sin [B (x - C)] + D$ .



$y =$  \_\_\_\_\_

**Verify the following identity.**

5)

$$\frac{1 - \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 - \sin \theta} = 2 \sec \theta$$

6) Find the exact value of each expression or state that it is undefined.

A)  $\cot(2\pi) =$

B)  $\sec\left(\frac{2\pi}{3}\right) =$

C)  $\tan\left(\frac{11\pi}{6}\right) =$

D)  $\sin\left(\frac{3\pi}{4}\right) =$

E)  $\csc\left(\frac{\pi}{2}\right) =$

Find all real numbers in radians that satisfy the equation.

7)

$$\tan 5x = -\sqrt{3}$$

**Find the exact value of each expression in radians.**

8) A)  $\csc^{-1}(2)$

B)  $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

C)  $\cot^{-1}(-\sqrt{3})$

9) **Find the amplitude, period, phase shift, and frequency of the given function.**

$$y = 6 \sin\left(5x + \frac{\pi}{2}\right)$$

Amplitude = \_\_\_\_\_

Period = \_\_\_\_\_

Phase shift = \_\_\_\_\_

Frequency = \_\_\_\_\_

**Find an equivalent algebraic expression for the composition.**

10)  $\cot(\arcsin(x))$

Name: \_\_\_\_\_ Instructor: \_\_\_\_\_

---

**This exam has three parts. Please read carefully the directions for each part. All problems are of equal point value. No notes, books, cell phones, or any device that can connect to the internet is allowed.**

---

## **PART TWO**

A NON-computer algebra system calculator is allowed. When directions specify an "exact value", a calculator should not be used.

Work all of the following problems. For full credit you must show all appropriate work and clearly indicate your answers.

---

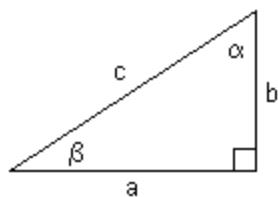
**Solve the problem. Round your answer to the nearest tenth.**

- 11) A force of 620 lb is required to pull a boat up a ramp inclined at  $16.0^\circ$  with the horizontal. How much does the boat weigh? Your work must include a sketch showing the given situation.

**Solve the triangle. If there is more than one triangle with the given parts, give both solutions. Round answers to the nearest tenth.**

12)  $\beta = 31.2^\circ$   
 $b = 9.7$   
 $a = 11.8$

**Solve the right triangle using the information given. Round answers to two decimal places, if necessary.**



13)  $a = 2, c = 8$ ; find  $b, \alpha$ , and  $\beta$

**Perform the indicated operation. Use the form  $\langle a, b \rangle$  for vectors.**

14) Given the vectors  $\mathbf{u} = \langle 11, -4 \rangle$ , and  $\mathbf{v} = \langle -10, 1 \rangle$

A) Find  $2\mathbf{u} - \mathbf{v}$

B) Find  $\mathbf{u} \cdot \mathbf{v}$

**Find all specified roots. Write your answers in  $a + bi$  form.**

15) Cube roots of 8.



**Solve the problem. Round your answers to the nearest tenth.**

16) A wheel is rotating at 15 radians/sec, and the wheel has a 33-inch diameter.

a) What is the angular velocity of a point on the rim in revolutions per minute?

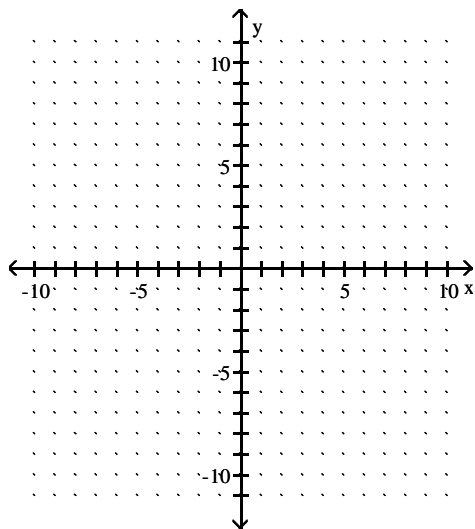
b) What is the linear velocity of a point on the rim in miles per hour? Note: there are 5280 feet in 1 mile.

**Use De Moivre's theorem to simplify the expression. Write the answer in a + bi form.**

17)  $(1 + i)^5$

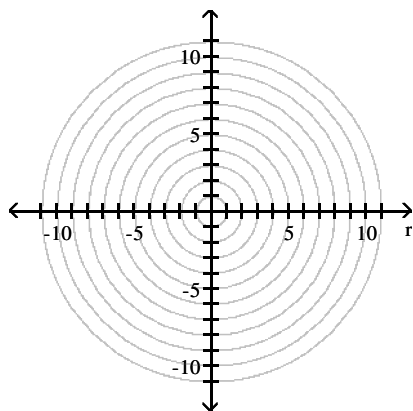
Graph the pair of parametric equations in the rectangular coordinate system. List at least 3 exact points that lie on the graph.

18)  $x = 3t - 1, y = 2 - t; -2 \leq t \leq 1$



Graph the polar equation. List at least four exact  $(r, \theta)$  points that lie on the graph.

19)  $r = 4 + 4 \cos \theta$



**Solve the problem. Round results to the nearest tenth.**

- 20) An airplane flies on a compass heading of  $90.0^\circ$  at 350 mph. The wind affecting the plane is blowing from a compass heading of  $311^\circ$  at 43.0 mph. What is the true course and ground speed of the airplane? Your work must include a sketch showing the given vectors and the resultant vector. Recall that compass headings are measured clockwise from due north.

## PART THREE

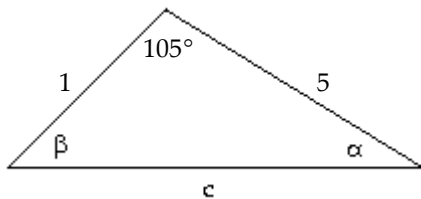
Choose any **THREE** of the following problems. Cross out the problems that you do not want to be graded. A NON-computer algebra system calculator is allowed. When directions specify an "exact value", a calculator should not be used.

For full credit you must show all appropriate work and clearly indicate your answers.

---

Find the area of the triangle. If necessary, round the answer to two decimal places.

21)



Verify the identity.

22)

$$\frac{\cos x}{1 + \sin x} = \sec x - \tan x$$

**For the given polar equation, write an equivalent rectangular equation.**

23)  $r = 10 \sin \theta$

**Solve the triangle. If there is more than one triangle with the given parts, give both solutions.  
Round answers to the nearest tenth.**

24)  $a = 8.2$   
 $b = 9.7$   
 $c = 12.5$

**Eliminate the parameter of the pair of parametric equations.**

25)  $x = t + 4, y = t^2$

**Solve the problem.**

- 26) The angle of elevation from a point on the ground to the top of a tower is  $35^\circ 42'$ . The angle of elevation from a point 112 feet farther back from the tower is  $22^\circ 13'$ . Find the height of the tower (to the nearest foot).

Name: Key Instructor: \_\_\_\_\_

This exam has three parts. Please read carefully the directions for each part. All problems are of equal point value. No notes, books, cell phones, or any device that can connect to the internet is allowed.

### PART ONE

You must complete this portion of the test **without** using a calculator. For full credit you must show all appropriate work and clearly indicate your answers. After you have finished part one, your instructor will give you the remaining parts of the exam.

When simplifying answers, it is **not** necessary to rationalize the denominator.

- 1) Given that  $\alpha$  is an angle in standard position whose terminal side contains the point  $(3, -2)$ , provide the exact value of each trigonometric function. It is not necessary to rationalize denominators.

$$\sin \alpha = -\frac{2}{\sqrt{13}}$$

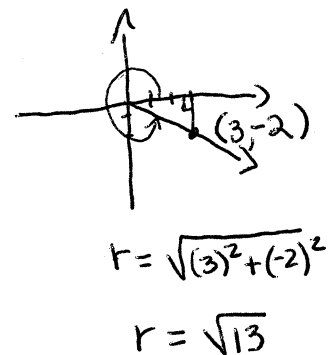
$$\cos \alpha = \frac{3}{\sqrt{13}}$$

$$\tan \alpha = -\frac{2}{3}$$

$$\csc \alpha = -\frac{\sqrt{13}}{2}$$

$$\sec \alpha = \frac{\sqrt{13}}{3}$$

$$\cot \alpha = -\frac{3}{2}$$



Find all real numbers in the interval  $[0, 2\pi)$  that satisfy the equation.

2)  $5 \cos x = 2 \cos^2 x + 2$

$$0 = 2 \cos^2 x - 5 \cos x + 2$$

$$0 = (2 \cos x - 1)(\cos x - 2)$$

$$2 \cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

OR  $\cos x - 2 = 0$

$$\cos x = 2$$

$$\boxed{x = \frac{\pi}{3} \text{ or } x = \frac{5\pi}{3}}$$

Solve the problem.

- 3) If  $\cos \alpha = \frac{2}{3}$  and  $\sin \beta = \frac{1}{5}$ , where  $0 \leq \alpha \leq \frac{\pi}{2}$  and  $\frac{\pi}{2} \leq \beta \leq \pi$ , then find  $\cos(\alpha - \beta)$ .

$$\left(\frac{2}{3}\right)^2 + \sin^2 \alpha = 1$$

$$\sin^2 \alpha = \frac{5}{9}$$

$$\alpha \text{ in QI: } \sin \alpha = \frac{\sqrt{5}}{3}$$

$$\left(\frac{1}{5}\right)^2 + \cos^2 \beta = 1$$

$$\cos^2 \beta = \frac{24}{25}$$

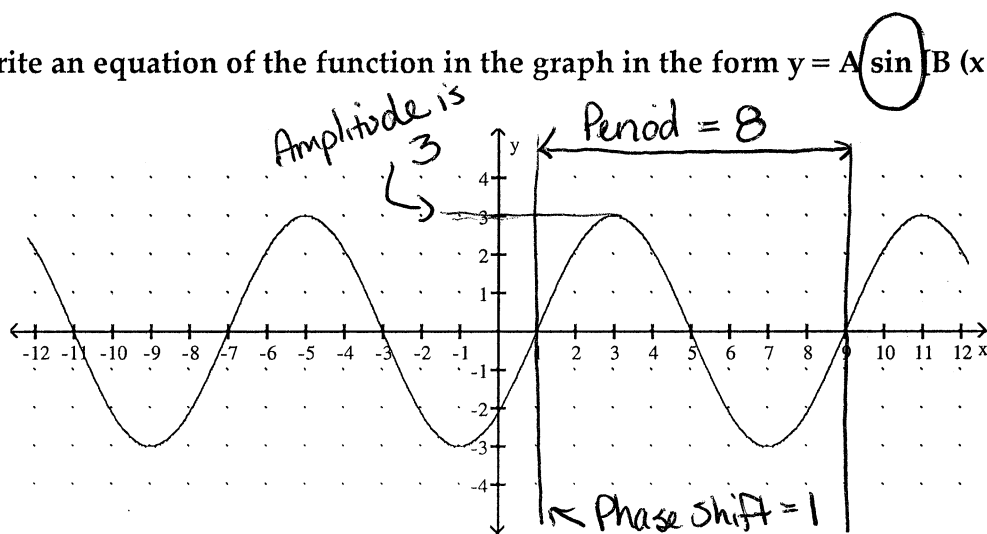
$$\beta \text{ in QII: } \cos \beta = -\frac{2\sqrt{6}}{5}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$= \left(\frac{2}{3}\right)\left(-\frac{2\sqrt{6}}{5}\right) + \left(\frac{\sqrt{5}}{3}\right)\left(\frac{1}{5}\right)$$

$$= \boxed{\frac{-4\sqrt{6} + \sqrt{5}}{15}}$$

- 4) Write an equation of the function in the graph in the form  $y = A \sin [B(x - C)] + D$ .



$$y = 3 \sin \left[ \frac{\pi}{4}(x - 1) \right]$$

$$B = \frac{2\pi}{8} = \frac{\pi}{4}$$

Other answers are acceptable.  
(For example,

$$y = -3 \sin \left[ \frac{\pi}{4}(x + 3) \right])$$

but it must be a sine function!



Verify the following identity.

5)

$$\frac{1 - \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 - \sin \theta} = 2 \sec \theta$$

$$\frac{1 - \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 - \sin \theta}$$

$$= \frac{(1 - \sin \theta)(1 - \sin \theta)}{\cos \theta (1 - \sin \theta)} + \frac{\cos \theta (\cos \theta)}{(1 - \sin \theta) (\cos \theta)}$$

Rewriting each term to have a common denominator

$$= \frac{1 - 2\sin \theta + \sin^2 \theta + \cos^2 \theta}{\cos \theta (1 - \sin \theta)}$$

Adding

$$= \frac{1 - 2\sin \theta + 1}{\cos \theta (1 - \sin \theta)}$$

Apply the Fundamental Pythagorean identity

$$= \frac{2 - 2\sin \theta}{\cos \theta (1 - \sin \theta)}$$

Combine like terms

$$= \frac{2(1 - \sin \theta)}{\cos \theta (1 - \sin \theta)}$$

Factor numerator and remove a factor of "one"

$$= \frac{2}{\cos \theta} = 2 \sec \theta$$

Basic identity for secant

Q.E.D.

6) Find the exact value of each expression or state that it is undefined.

A)  $\cot(2\pi) = \boxed{\text{undefined}}$

$(\sin(2\pi) = 0)$

B)  $\sec\left(\frac{2\pi}{3}\right) = \frac{1}{\cos\left(\frac{2\pi}{3}\right)} = -2$

C)  $\tan\left(\frac{11\pi}{6}\right) = \frac{\sin\left(\frac{11\pi}{6}\right)}{\cos\left(\frac{11\pi}{6}\right)} = \boxed{-\frac{1}{\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3}}$

D)  $\sin\left(\frac{3\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{2}} \approx \frac{1}{\sqrt{2}}$

E)  $\csc\left(\frac{\pi}{2}\right) = \frac{1}{\sin\left(\frac{\pi}{2}\right)} = \boxed{1}$

Find all real numbers in radians that satisfy the equation.

7)

$$\tan 5x = -\sqrt{3}$$

$$5x = \frac{2\pi}{3} + K\pi$$

$$\boxed{x = \frac{2\pi}{15} + \frac{K\pi}{5}}$$

Find the exact value of each expression in radians.

8) A)  $\csc^{-1}(2) = \sin^{-1}(\frac{1}{2}) = \boxed{\frac{\pi}{6}}$   $\star -\frac{\pi}{2} \leq \sin^{-1}(x) \leq \frac{\pi}{2}$

B)  $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{\frac{3\pi}{4}}$   $\star 0 \leq \cos^{-1}(x) \leq \pi$

C)  $\cot^{-1}(-\sqrt{3}) = \boxed{\frac{5\pi}{6}}$

$\frac{\cos \theta}{\sin \theta} = -\sqrt{3}$   
 $0 < \theta < \pi$

9) Find the amplitude, period, phase shift, and frequency of the given function.

$y = 6 \sin\left[5x + \frac{\pi}{2}\right] = 6 \sin\left[5\left(x + \frac{\pi}{10}\right)\right]$

Amplitude = 6

Period =  $\frac{2\pi}{5}$

Phase shift =  $-\frac{\pi}{10}$

Frequency =  $\frac{5}{2\pi}$

$\star \text{ Period} = \frac{2\pi}{B}$

$\star \text{ Frequency} = \frac{1}{\text{Period}}$

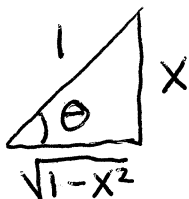
Find an equivalent algebraic expression for the composition.

10)  $\cot(\arcsin(x))$

$\cot(\arcsin(x))$

$\arcsin(x) = \theta$

$= \cot(\theta)$



$= \boxed{\frac{\sqrt{1-x^2}}{x}}$

Name: \_\_\_\_\_ Instructor: \_\_\_\_\_

This exam has three parts. Please read carefully the directions for each part. All problems are of equal point value. No notes, books, cell phones, or any device that can connect to the internet is allowed.

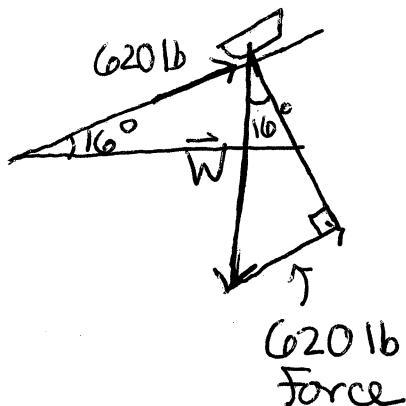
## PART TWO

A NON-computer algebra system calculator is allowed. When directions specify an "exact value", a calculator should not be used.

Work all of the following problems. For full credit you must show all appropriate work and clearly indicate your answers.

Solve the problem. Round your answer to the nearest tenth.

- 11) A force of 620 lb is required to pull a boat up a ramp inclined at  $16.0^\circ$  with the horizontal. How much does the boat weigh? Your work must include a sketch showing the given situation.



$|\vec{W}|$  is the weight of the boat

$$\sin 16^\circ = \frac{620}{|\vec{W}|}$$

$$|\vec{W}| = \frac{620}{\sin 16^\circ}$$

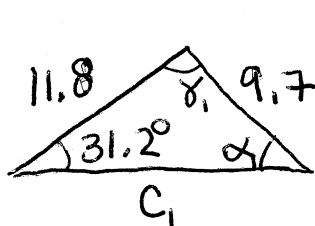
The boat weighs  $\approx 2249.3$  lb

Solve the triangle. If there is more than one triangle with the given parts, give both solutions. Round answers to the nearest tenth.

12)  $\beta = 31.2^\circ$   
 $b = 9.7$   
 $a = 11.8$

$$\frac{\sin 31.2^\circ}{9.7} = \frac{\sin \alpha}{11.8}$$

Triangle 1:  
 $\alpha_1 = 39.1^\circ, \gamma_1 = 109.7^\circ, c_1 = 17.6$



$$\alpha_1 = \sin^{-1}\left(\frac{11.8 \sin 31.2^\circ}{9.7}\right) \approx 39.1^\circ$$

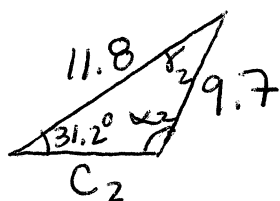
$$\gamma_1 = 180^\circ - (31.2^\circ + 39.1^\circ) = 109.7^\circ$$

$$\alpha_2 = 180^\circ - 39.1^\circ = 140.9^\circ$$

$$c_1 = \frac{9.7 \sin 109.7^\circ}{\sin 31.2^\circ}$$

$$c_1 \approx 17.6$$

possible to get second triangle with  $\beta = 31.2^\circ$



$$c_2 = \frac{9.7 \sin 7.9^\circ}{\sin 31.2^\circ}$$

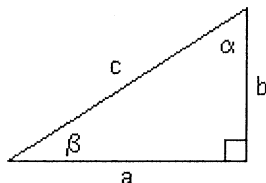
$$c_2 \approx 2.6$$

Triangle 2:  
 $\alpha_2 = 140.9^\circ, \gamma_2 = 7.9^\circ, c_2 = 2.6$

$$\gamma_2 = 180^\circ - (31.2^\circ + 140.9^\circ)$$

$$\gamma_2 = 7.9^\circ$$

Solve the right triangle using the information given. Round answers to two decimal places, if necessary.



13)  $a = 2, c = 8$ ; find  $b, \alpha$ , and  $\beta$

$$2^2 + b^2 = 8^2$$

$$b^2 = 60$$

$$b = \sqrt{60}$$

$$b = 2\sqrt{15}$$

$$b \approx 7.75$$

$$\cos \beta = \frac{2}{8}$$

$$\cos \beta = \frac{1}{4}$$

$$\beta = \cos^{-1}\left(\frac{1}{4}\right)$$

$$\beta \approx 75.52^\circ$$

$$\alpha = 90^\circ - 75.52^\circ$$

$$\alpha = 14.48^\circ$$

Perform the indicated operation. Use the form  $\langle a, b \rangle$  for vectors.

- 14) Given the vectors  $\mathbf{u} = \langle 11, -4 \rangle$ , and  $\mathbf{v} = \langle -10, 1 \rangle$

A) Find  $2\mathbf{u} - \mathbf{v}$

$$2\vec{u} - \vec{v} = 2\langle 11, -4 \rangle - \langle -10, 1 \rangle \\ = \langle 22, -8 \rangle - \langle -10, 1 \rangle \rightarrow = \boxed{\langle 32, -9 \rangle}$$

B) Find  $\mathbf{u} \cdot \mathbf{v}$

$$\vec{u} \cdot \vec{v} = \langle 11, -4 \rangle \cdot \langle -10, 1 \rangle \\ = (11)(-10) + (-4)(1) \\ = \boxed{-114}$$

Find all specified roots. Write your answers in  $a + bi$  form.

- 15) Cube roots of 8.

$$8 = 8(\cos(0^\circ) + i\sin(0^\circ))$$

1st cube root:  $\sqrt[3]{8}(\cos(\frac{0^\circ}{3}) + i\sin(\frac{0^\circ}{3}))$

$$= 2(1 + 0i)$$

$$= \boxed{2 + 0i \text{ or } 2}$$

2nd cube root:  $\sqrt[3]{8}(\cos(120^\circ) + i\sin(120^\circ))$

$$= 2(-\frac{1}{2} + \frac{\sqrt{3}}{2}i)$$

$$= \boxed{-1 + \sqrt{3}i}$$

[Note: cube roots  $\Rightarrow$

Add  $\frac{360^\circ}{3} = 120^\circ$

to  $0^\circ$  to get second angle;

add  $2(120^\circ) = 240^\circ$  for

third]

3rd cube root:  $\sqrt[3]{8}(\cos(240^\circ) + i\sin(240^\circ))$

$$= 2(-\frac{1}{2} - \frac{\sqrt{3}}{2}i)$$

$$= \boxed{-1 - \sqrt{3}i}$$

Solve the problem. Round your answers to the nearest tenth.

16) A wheel is rotating at 15 radians/sec, and the wheel has a 33-inch diameter.

a) What is the angular velocity of a point on the rim in revolutions per minute?

$$\frac{15 \text{ radians}}{\text{seconds}} \cdot \frac{1 \text{ revolution}}{2\pi \text{ radians}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} = \frac{450}{\pi} \frac{\text{revolutions}}{\text{minute}}$$

$$\approx 143.2 \text{ rev/min}$$

b) What is the linear velocity of a point on the rim in miles per hour? Note: there are 5280 feet in 1 mile.

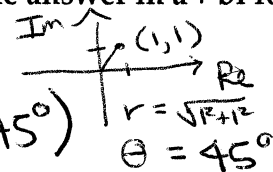
$$V = \omega r$$

$$V = \left( \frac{15 \text{ rad}}{\text{sec}} \cdot \frac{3600 \text{ sec}}{\text{hr}} \right) \cdot \left( \frac{33 \text{ in}}{2} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} \right) \approx 14.1 \frac{\text{miles}}{\text{hr}}$$

$\omega$  in rad/hr                       $r$  in miles

Use De Moivre's theorem to simplify the expression. Write the answer in a + bi form.

17)  $(1+i)^5$

$$Z = 1+i = \sqrt{2} (\cos 45^\circ + i \sin 45^\circ)$$


$r = \sqrt{1^2 + 1^2} = \sqrt{2}$   
 $\theta = 45^\circ$

$$Z^5 = (\sqrt{2})^5 (\cos(5 \cdot 45^\circ) + i \sin(5 \cdot 45^\circ))$$

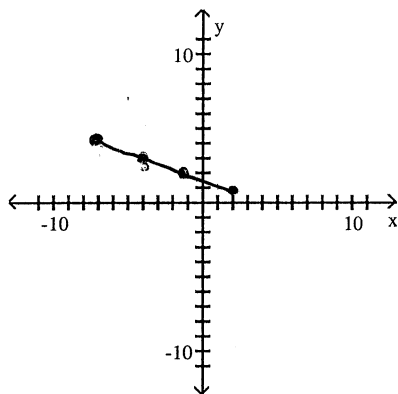
$$Z^5 = 4\sqrt{2} (\cos(225^\circ) + i \sin(225^\circ))$$

$$Z^5 = 4\sqrt{2} \left( -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i \right)$$

$$Z^5 = \boxed{-4 - 4i}$$

Graph the pair of parametric equations in the rectangular coordinate system. List at least 3 exact points that lie on the graph.

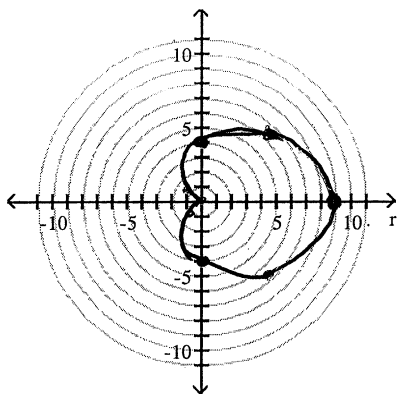
18)  $x = 3t - 1, y = 2 - t; -2 \leq t \leq 1$



t	x	y
-2	-7	4
-1	-4	3
0	-1	2
1	2	1

Graph the polar equation. List at least four exact  $(r, \theta)$  points that lie on the graph.

19)  $r = 4 + 4 \cos \theta$

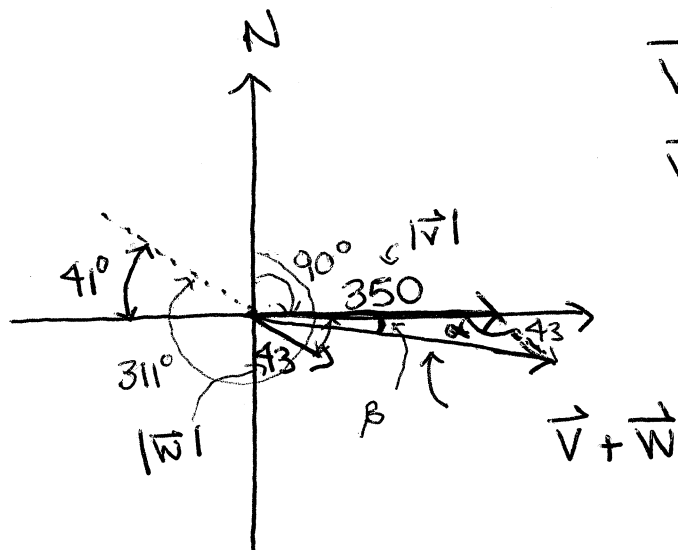


$\theta$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	$\pi$	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	$2\pi$
r	8	↑	4	↑	0	↑	4	↑	8
	$4+2\sqrt{2}$		$4-2\sqrt{2}$		$4-2\sqrt{2}$		$4+2\sqrt{2}$		$4+2\sqrt{2}$



Solve the problem. Round results to the nearest tenth.

- 20) An airplane flies on a compass heading of  $90.0^\circ$  at 350 mph. The wind affecting the plane is blowing from a compass heading of  $311^\circ$  at 43.0 mph. What is the true course and ground speed of the airplane? Your work must include a sketch showing the given vectors and the resultant vector. Recall that compass headings are measured clockwise from due north.



$\vec{V}$  = velocity vector for plane

$\vec{W}$  = velocity vector for wind

$$\alpha = 180^\circ - 41^\circ = 139^\circ$$

$$|\vec{V} + \vec{W}|^2 = (350)^2 + (43)^2 - 2(350)(43)\cos(139^\circ)$$

$$|\vec{V} + \vec{W}| \approx 383.5 \text{ mph is the ground speed}$$

True Course :  $90^\circ + \text{drift angle}$

drift angle : " $\beta$ "

$$\frac{\sin \beta}{43} = \frac{\sin 139^\circ}{383.5}$$

$$\beta = \sin^{-1}\left(\frac{43 \sin 139^\circ}{383.5}\right)$$

$$\beta \approx 4.2^\circ$$

$$= 90^\circ + 4.2^\circ$$

$$= 94.2^\circ$$

True course is bearing  $94.2^\circ$

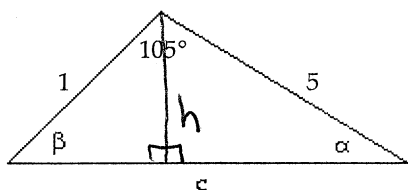
### PART THREE

Choose any **THREE** of the following problems. Cross out the problems that you do not want to be graded. A NON-computer algebra system calculator is allowed. When directions specify an "exact value", a calculator should not be used.

For full credit you must show all appropriate work and clearly indicate your answers.

Find the area of the triangle. If necessary, round the answer to two decimal places.

21)



$$\sin \beta = \frac{h}{1}$$

$$\frac{\sin 105^\circ}{5} = \frac{\sin \beta}{1} \Rightarrow h = \frac{5 \sin 105^\circ}{1}$$

$$A = \frac{1}{2} \text{ base} \cdot \text{height}$$

$$A = \frac{1}{2} \cancel{c} \left( \frac{5 \sin 105^\circ}{\cancel{c}} \right)$$

$$A = \frac{1}{2} 5 \sin 105^\circ$$

$$\boxed{A \approx 2.41}$$

Verify the identity.

22)

$$\frac{\cos x}{1 + \sin x} = \sec x - \tan x$$

$$\sec x - \tan x = \frac{1}{\cos x} - \frac{\sin x}{\cos x}$$

$$= \frac{1 - \sin x}{\cos x}$$

$$= \frac{(1 - \sin x)(1 + \sin x)}{\cos x (1 + \sin x)}$$

$$= \frac{1 - \sin^2 x}{\cos x (1 + \sin x)}$$

$$= \frac{\cos^2 x}{\cos x (1 + \sin x)}$$

$$= \frac{\cos x}{1 + \sin x} \quad \text{QED}$$

OR

$$\frac{\cos x}{1 + \sin x} = \frac{\cos x (1 - \sin x)}{(1 + \sin x)(1 - \sin x)}$$

$$= \frac{\cos x (1 - \sin x)}{1 - \sin^2 x}$$

$$= \frac{\cos x (1 - \sin x)}{\cos^2 x}$$

$$= \frac{1 - \sin x}{\cos x}$$

$$= \frac{1}{\cos x} - \frac{\sin x}{\cos x}$$

$$= \sec x - \tan x \quad \text{QED}$$

For the given polar equation, write an equivalent rectangular equation.

23)  $r = 10 \sin \theta$

$$r \cdot (r) = r(10 \sin \theta)$$

$$r^2 = 10r \sin \theta$$

$$\boxed{x^2 + y^2 = 10y}$$

Solve the triangle. If there is more than one triangle with the given parts, give both solutions. Round answers to the nearest tenth.

24)  $a = 8.2$

$b = 9.7$

$c = 12.5$

$$(12.5)^2 = (8.2)^2 + (9.7)^2 - 2(8.2)(9.7)\cos \gamma$$

$$\gamma = \cos^{-1} \left( \frac{(12.5)^2 - (8.2)^2 - (9.7)^2}{-2(8.2)(9.7)} \right) \approx 88.2^\circ$$

$$\frac{\sin \alpha}{8.2} = \frac{\sin 88.2^\circ}{12.5}$$

$$\alpha = \sin^{-1} \left( \frac{8.2 \sin 88.2^\circ}{12.5} \right)$$

$$\alpha \approx 41.0^\circ$$

$$\beta = 180^\circ - (88.2^\circ + 41.0^\circ)$$

$$\beta = 50.8^\circ$$

(rounded)

$$\boxed{\begin{array}{l} \alpha = 41.0^\circ \\ \beta = 50.8^\circ \\ \gamma = 88.2^\circ \end{array}}$$

Eliminate the parameter of the pair of parametric equations.

25)  $x = t + 4, y = t^2$

$$x = t + 4$$

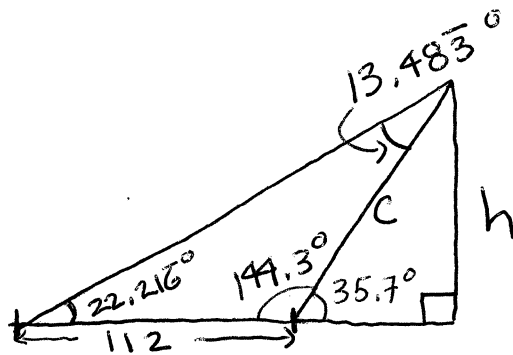
$$\Rightarrow t = x - 4$$

$$y = t^2$$

$$y = (x - 4)^2$$

Solve the problem.

- 26) The angle of elevation from a point on the ground to the top of a tower is  $35^\circ 42'$ . The angle of elevation from a point 112 feet farther back from the tower is  $22^\circ 13'$ . Find the height of the tower (to the nearest foot).



$$35^\circ 42' = 35.7^\circ$$

$$22^\circ 13' = 22.216^\circ$$

$$\frac{C}{\sin(22.216^\circ)} = \frac{112}{\sin(13.483^\circ)}$$

$$C = \frac{112 \sin(22.216^\circ)}{\sin(13.483^\circ)}$$

$$C \approx 181.6 \text{ ft}$$

$$\sin 35.7^\circ = \frac{h}{181.6}$$

$$181.6 \sin 35.7^\circ = h$$

$$h \approx 106 \text{ ft}$$